STUDY OF A LATTICE DYNAMIC SUBMITTED TO A LOCAL NOISE PRESERVING THE ENERGY

Gaëtan Cane
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PSPDE-IX

Presentation of the dynamic

Let $(\eta(x))_{x\in\mathbb{Z}}$ be a configuration defined as follows:

$$\forall x \in \mathbb{Z}, \quad \forall t \in [0,T], \quad \eta_t(x) = \eta_0(x) + \int_0^t (\eta_s(x+1) - \eta_s(x-1))ds.$$

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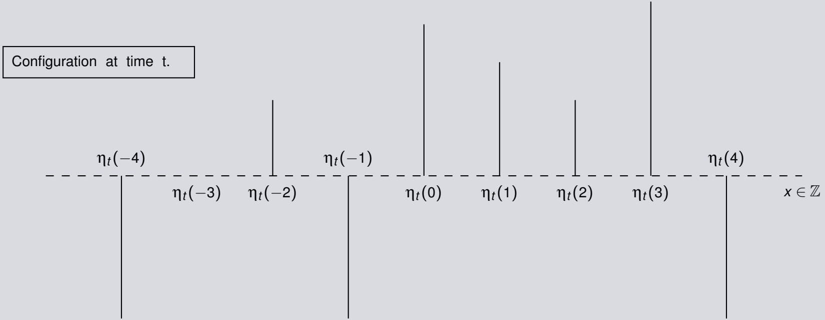

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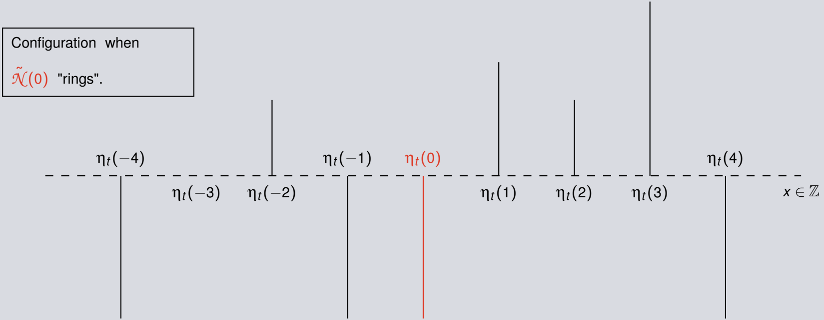

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LOCAL NOISE IS A PROBLEM !

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We introduce a smooth function γ with compact support $[-1, 1]$.

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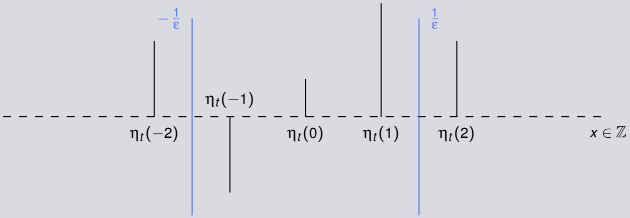

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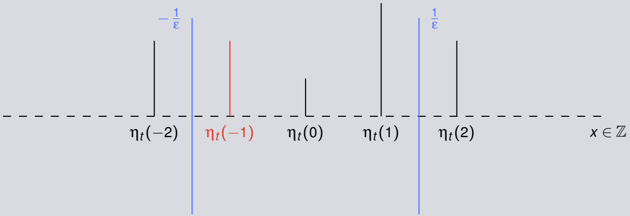

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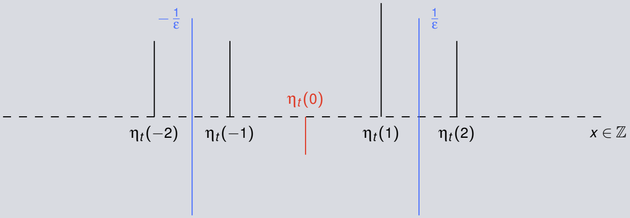

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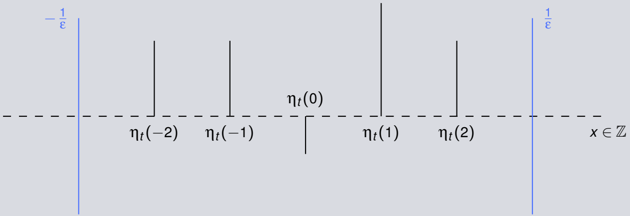

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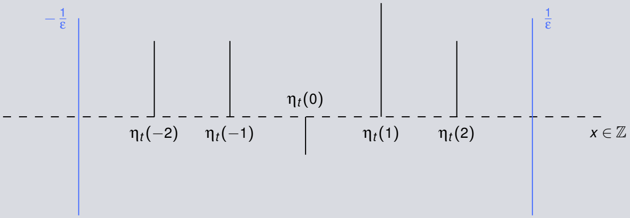

The dynamic is generated by the generator $\mathcal{L}$ where for $\phi \in \mathcal{D}(\mathcal{L})$ we have:

$$\mathcal{L}[\phi] = \mathcal{A}[\phi] + \varepsilon \mathcal{S}^{\text{flip}}[\phi] \quad \varepsilon > 0,$$

with:

$$\mathcal{S}^{\text{flip}}[\phi(\eta)] = \sum_{x \in \mathbb{Z}} \gamma(\varepsilon x)\left[\phi\left(\eta^x\right) - \phi\left(\eta\right)\right].$$

Let $T > 0$ and $\mu \in \mathcal{C}\left([0,T], \mathcal{M}_b^+\right)$ such that:

$$\forall t \in [0,T],\ \langle \mu(t), J\rangle - \langle \mu(0), J\rangle = \int_0^t \frac{1}{2\pi}\left\langle \mu(s), \partial_k \omega \partial_u J\right\rangle ds + \int_0^t \left\langle \mu(s), C_\gamma J\right\rangle ds,$$

where $\mu(0) = \mu_0$ and

$$CJ(u,k) = 4\gamma(u)\left[\int_{\mathbb{T}} \left(\bar{J}(u,k') - \bar{J}(u,k)\right) dk'\right], \quad \omega(k) = \sin(2\pi k).$$

Kinetic limit of $\mathcal{W}_{\mu_\varepsilon}(\cdot)$

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Theorem

Under some assumptions, $(\mathcal{W}_{\mu_\varepsilon}(\cdot))_{\varepsilon>0}$ *converges pointwise in* $\mathcal{C}\left([0,T], \mathcal{M}_b^+\right)$ *to* $(\mu_t)_{t\in[0,T]}$.

Introduction of a PDMP

$$\forall u \in \mathbb{R}, \quad \gamma_\delta(u) = \frac{\gamma(\frac{u}{\delta})}{\delta}\delta^{1-\beta} \quad \text{with} \quad \beta \geq 0.$$

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If we assume that $\mu^\delta(\cdot)$ has a density $f^\delta(\cdot)$ then:

$$\partial_t f^\delta(u,k,t) = -\frac{1}{2\pi}\cos(2\pi k)\partial_u f^\delta(u,k,t) + 4\gamma_\delta(u)\int_{\mathbb{T}}\left(f^\delta(u,k',t) - f^\delta(u,k,t)\right)dk',$$

with $f^\delta(u,k,0) = f_0(u,k)$ and $(u,k) \in \mathbb{R}\times\mathbb{T}$.

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We conclude that:

$$\forall t \in [0,T], \quad \forall (u,k) \in \mathbb{R}\times\mathbb{T}, \quad f^\delta(u,k,t) = \mathbb{E}_{(u,k)}\left[f_0(U_t^\delta, K_t^\delta)\right],$$

where $(U_t^\delta, K_t^\delta)_{t\geq 0}$ is a PDMP.

- $\beta < 1$: $\lim_{\delta \to 0} f^{\delta}(u, k, t) = f_0(u - v(k)t)$.

Study of the case $\beta \leq 1$

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